Activity description

This activity is about using graphical and algebraic methods to solve problems in real contexts that can be modelled using quadratic expressions. The activity could be used in many different ways, depending on the needs of the students.

Suitability and Time

Level 2 (Higher) or Level 3 (Advanced)
2-4 hours, depending on student prior knowledge and how the resource is used.

Resources

Student information sheet and worksheet
Optional: slideshow

Equipment

Access to spreadsheet or graphing software.
Graph paper if graphs are to be plotted manually.

Key mathematical language

Area of cross-section, radius, quadratic, quadratic formula, solution

Notes on the activity

This gives two methods for solving a problem involving a quadratic function. You could use both methods together, or split the materials and use the two methods at different times in the course. Using both methods together helps students to see the connections between mathematical topics.

The first method involves using Excel to draw a quadratic graph. Students will need to be familiar with the use of 'functions' and 'fill down' in Excel, and also know how to format graphs.

The second method involves the use of the quadratic formula. This could be used as a follow-up to the Nuffield resource 'The quadratic formula'.

The worksheet of mixed problems can be solved using either method. Allowing students to choose their own method helps them to develop independent problem-solving skills.

During the activity

Students could discuss the mathematics and work together collaboratively. For example, pairs of students could do the same problem but one could solve it algebraically and the other graphically, comparing methods and answers.

Points for discussion

If students use both methods, ask them which they prefer and why.
Discuss the advantages and disadvantages of each method, where errors may occur, and how accurate the final answer will be.

Extensions

Students could be asked to imagine that the designer has asked them to give minimum and maximum values for the radius *r*, but unfortunately they have neither graph paper nor calculator with them. How accurate an answer can be achieved just using a paper and pencil? Is it accurate enough for the designer to use in his design?

Answers

Using the graph

Minimum road width is 4.3 m (to 1 dp).

Maximum value of $r \approx 3.42$ metres, giving a maximum road width of 6.8 metres (to 1 dp).

Using the quadratic formula

For an area of 32 m^2 ,

$0.5\pi r^2 + 4r = 32$, which rearranges to $0.5\pi r^2 + 4r - 32 = 0$

In the quadratic formula: $a = 0.5\pi$, $b = 4$, $c = -32$

Using the formula: $r = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 0.5 \times \pi \times -32}}{2 \times 0.5 \times \pi}$

$$r = \frac{-4 \pm \sqrt{217.06}}{\pi} = \frac{-4 \pm 14.733}{\pi}$$

The radius must be positive so $r = \dfrac{-4 + 14.733}{\pi} = \dfrac{10.733}{\pi} = 3.416$

Tunnel width = $2r = 2 \times 3.416 = 6.83$,
so maximum width = 6.8 metres to 1 dp.

Mixed problems

1 x = 1.5 m (to 1 dp)

2 Radius = 3.2 cm (to 1 dp)

3 Radius = 9.4 cm (to 1 dp)

4a 7.6 cm (to 1 dp) **b** 11.6 cm (to 1 dp.)

5 r = 1.9 cm (to 1 dp)